Experiment 10
Moment of Inertia
(Fall 2020 version)

Advanced Reading

Moment of Inertia section of text

Equipment

- •Beck's Inertia Thing (rotational apparatus)
- Vernier caliper
- masses
- meter stick
- stopwatch (cell phones are ok to use)

Objective

The objective of this experiment is to dynamically measure the moment of inertia of a rotating system and to compare this to a predicted value.

Theory

The moment of inertia can be viewed as the rotational analog of *mass.* Torque and angular acceleration are the rotational analogs of *force* and *acceleration*, respectively. Thus, in rotational dynamics, Newton's second Law (F=ma) becomes $\tau = I\alpha$, where τ is the (net) applied torque, I is the moment of inertia of the body and α is the angular acceleration.

An object that experiences constant angular acceleration must have a constant torque applied to it. By applying a known torque to a rigid body, measuring the angular acceleration, and using the relationship $\tau = I\alpha$, the moment of inertia I can be found.

In this experiment, a torque is applied to the rotational apparatus by a string that is wrapped around the axle of the apparatus (Fig. 9-1).

Figure 9-1

The tension T is supplied by a hanging weight mg. The tension is found by applying Newton's second law (Fig. 9-2).

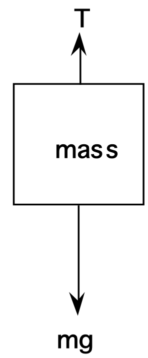

Figure 9-2 Forces on the hanging mass.

If we take upward direction as positive, and apply Newton's second law, we have

$$\Sigma F = T - mg = -ma\ ,$$

so the tension is

$$T = m(g - a)$$

The rotational apparatus has an original moment of inertia I_O with no additional masses added. When additional masses are added, it has a new moment of inertia I_{new}. The relationship between I_o and I_{new} is given by $I_{new} = I_o + MR^2$ where M is the total added mass and R is the distance of this mass from the center of the wheel (i.e., from the axis of rotation). *). Please note that it is assumed that the added masses are point masses. We will explore whether this is an appropriate assumption in the questions.*

Procedure

Part 1. Moment of Inertia of apparatus without additional weights

1. Using the vernier caliper, measure the diameter of the axle around which the string wraps. Calculate the radius r_{axle}. Make sure that no additional masses are added to the apparatus (Fig. 9-1).

2. Wrap the string around the axle and place enough weight on the string to cause the apparatus to rotate at a constant speed (or very slowly). The angular acceleration should be nearly zero. When

this is the case, the sum of the torques on the body must be (nearly) zero. The amount of mass needed is on the order of a few grams. From this data, calculate the frictional torque which is given by

$$\tau_{friction} = r_{axle}F_{friction} = r_{axle}m_{friction}g \ .$$

3. REMOVE FRICTION MASS. Place a 50 gram mass hanger on string. Measure the distance from the bottom of the weight hanger to the floor. Release the weight hanger, being sure not to impart an initial velocity to the wheel of the rotational apparatus.

4. Use the stopwatch to time the fall. Perform a total of five trials and calculate an average distance and an average time. From this information, calculate the acceleration of the mass using $distance = 1/2at^2$. Calculate the angular acceleration $\alpha = a/r$, where r is the radius of the axle that the string is wrapped around.

5. Next, calculate the tension of the string. (See the Theory section.)

6. The applied torque on the spinning wheel is provided by the tension of the string. Use the value of the tension to calculate this torque. Next, **calculate the net torque**, which is the **applied torque minus the frictional torque**.

7. Add 50 grams to hanger **for a total of 100 grams.** Repeat steps 3-6.

8. **Plot the net torque vs. angular acceleration. Be sure to enter the origin as a data point**. Determine the moment of inertia I_o, which is the slope of the best-fit line.

Part II Additional masses

9. Measure the distance from the center of the inertia wheel to the center of the outer set of tapped-holes. Do this for all three arms and average the distance. (*They should all be nearly the same.*)

11. Add the total mass of the three brass masses. The mass of each one is written on the side and/or top. Attach the masses to the apparatus. Calculate the new moment of inertia, I_{new}, with these additional masses located at a distance R from the axis of rotation. **Measure the diameter of the masses. This information will be needed to answer question 3.**

12. Repeat the steps 2 through 8 for this new moment of inertia. Plot the data and determine moment of inertia I_{new} from the slope. Calculate the percent difference between the experimental value and the calculated value.

Questions/Conclusions

1. What is the maximum kinetic energy that the inertia device (wheel) used is given by the hanging mass (just before the mass reaches the floor). Compare this value to the gravitational energy that the hanging mass has just before you release it. **Choose only one of your 4 runs and say which you used.** Show all work.

2. If the torque applied to a rigid body is doubled, what happens to the moment of inertia of the body? What happens to the angular acceleration? Explain your answers.

3. In the theoretical determination of the moment of inertia I_{new} with the additional masses, it was assumed that the masses are points.

Using the parallel-axis theorem, calculate the moment of inertia such that the diameter of the masses is taken into account. Determine the percentage difference between this and the previous value. Is it a good approximation to assume that the masses are points in this particular case?

4. What is the moment of inertia of a drum which is rotated about the vertical axis which runs through the center of the drum. Assume drum is a hoop. You should ignore the top and bottom pieces of drum and assume drum is a hoop**. See table below for which drum you should use. Table from website https://www.skolnik.com/steel-drum-measurements**

Steel Drum Measurements

<table>
<tr><th>CAPACITY</th><th colspan="3">MEASUREMENTS</th><th colspan="2">MAX WEIGHT & GAUGE</th></tr>
<tr><th>Gallons | Liters</th><th>Inside</th><th>US</th><th>Metric</th><th>US</th><th>Metric</th></tr>
<tr><td rowspan="2">sec 1 3 | 11</td><td>Diameter</td><td>14"</td><td>36cm</td><td>8 lbs.</td><td>4 kg</td></tr>
<tr><td>Height</td><td>6"</td><td>15cm</td><td>20 gauge</td><td>0.9 mm</td></tr>
<tr><td rowspan="2">sec 2 5 | 19</td><td>Diameter</td><td>14"</td><td>36cm</td><td>10 lbs.</td><td>4.5 kg</td></tr>
<tr><td>Height</td><td>10"</td><td>25cm</td><td>20 gauge</td><td>0.9 mm</td></tr>
<tr><td rowspan="2">sec 3 8 | 30</td><td>Diameter</td><td>14.00"</td><td>35 cm</td><td>12 lbs.</td><td>5 kg</td></tr>
<tr><td>Height</td><td>14.00"</td><td>36 cm</td><td>20 gauge</td><td>0.90 mm</td></tr>
<tr><td rowspan="2">sec 4 10 | 38</td><td>Diameter</td><td>14.00"</td><td>35 cm</td><td>14 lbs.</td><td>6 kg</td></tr>
<tr><td>Height</td><td>17.00"</td><td>43 cm</td><td>20 gauge</td><td>0.90 mm</td></tr>
<tr><td rowspan="2">sec 5 16 | 60</td><td>Diameter</td><td>14"</td><td>36cm</td><td>19 lbs.</td><td>9 kg</td></tr>
<tr><td>Height</td><td>27"</td><td>70cm</td><td>20 gauge</td><td>0.9 mm</td></tr>
<tr><td rowspan="2">sec 6 20 | 76</td><td>Diameter</td><td>18.25"</td><td>46 cm</td><td>30 lbs.</td><td>14 kg</td></tr>
<tr><td>Height</td><td>19.25</td><td>49 cm</td><td>18 gauge</td><td>1.20 mm</td></tr>
<tr><td rowspan="2">sec 8 30 | 114</td><td>Diameter</td><td>18.25"</td><td>46 cm</td><td>35 lbs.</td><td>16 kg</td></tr>
<tr><td>Height</td><td>27.50"</td><td>70 cm</td><td>18 gauge</td><td>1.50 mm</td></tr>
<tr><td rowspan="2">sec 9 55 | 208</td><td>Diameter</td><td>22.50"</td><td>57 cm</td><td>60 lbs.</td><td>27 kg</td></tr>
<tr><td>Height</td><td>33.00"</td><td>84 cm</td><td>16 gauge</td><td>1.50 mm</td></tr>
<tr><td rowspan="2">85 | 322 sec 10</td><td>Diameter</td><td>26.00"</td><td>66 cm</td><td>80 lbs.</td><td>35 kg</td></tr>
<tr><td>Height</td><td>37.00"</td><td>92 cm</td><td>16 gauge</td><td>1.50 mm</td></tr>
<tr><td rowspan="2">96 | 363 sec 12</td><td>Diameter</td><td>26.00"</td><td>66 cm</td><td>85 lbs.</td><td>39 kg</td></tr>
<tr><td>Height</td><td>41.00"</td><td>104 cm</td><td>16 gauge</td><td>1.50 mm</td></tr>
<tr><td rowspan="2">sec 110 | 416 13</td><td>Diameter</td><td>30.00"</td><td>76 cm</td><td>102 lbs.</td><td>46 kg</td></tr>
<tr><td>Height</td><td>41.00"</td><td>104 cm</td><td>16 gauge</td><td>1.50 mm</td></tr>
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